MATH 122: CALCULUS I
SUPPLEMENTAL INSTRUCTION FOR
AT-RISK STUDENTS
Teaching Assistant Training Manual
Kagba Suaray
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INTRODUCTION: ABOUT MATH 122

PREREQUISITES

Appropriate MDPT placement or a grade of "C" or better in MATH 111 and MATH 113.

111. Precalculus Trigonometry (3 UNITS)
Prerequisite: Appropriate ELM score, ELM exemption, or MAPB 11.

Trigonometric functions and applications. Arithmetic and graphical representation of complex numbers, polar form, DeMoivre's Theorem. Not open for credit to students with credit in MATH 101, MATH 117 or MATH 122. (Lecture 3 hrs.)

113. Precalculus Algebra (3 UNITS)
Prerequisite: Appropriate ELM score, ELM exemption, or MAPB 11.

Equations, inequalities. Functions, their graphs, inverses, transformations. Polynomial, rational functions, theory of equations. Exponential, logarithmic functions, modeling. Systems of equations, matrices, determinants. Sequences, series. Not open for credit to students with credit in MATH 112, MATH 115, MATH 117, MATH 119A, MATH 120, or MATH 122. For students who will continue to MATH 115, MATH 119A, or MATH 122. (Lecture 3 hrs.)

CATALOG DESCRIPTION

122. Calculus I (4 UNITS)
Continuous functions. Derivatives and applications including graphing, related rates, and optimization. Transcendental functions. L'Hospital's Rule. Antiderivatives. Definite integrals. Area under a curve. (Lecture 3 hrs. problem session 2 hrs.)
# COURSE OUTLINE

**DEPARTMENT OF MATHEMATICS AND STATISTICS**  
**MATH 122 — CALCULUS I — COURSE OUTLINE**  
**Effective Fall 2012**


The outline is based on 13 weeks of lectures. This leaves approximately 2 weeks for leeway, exams, and review. The suggested times in the outline are approximate. Sections that are enclosed in parentheses may be abridged or omitted.

<table>
<thead>
<tr>
<th>Week</th>
<th>Sections</th>
<th>Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3, 1.4</td>
<td>Limits of functions, calculating limits</td>
</tr>
<tr>
<td>2</td>
<td>1.5, 1.6</td>
<td>Continuity, limits involving infinity, asymptotes</td>
</tr>
<tr>
<td>3</td>
<td>2.1, 2.2, 2.3</td>
<td>Definition of derivative, differentiation rules</td>
</tr>
<tr>
<td>4</td>
<td>2.4, 2.5, 2.6</td>
<td>More differentiation rules, implicit differentiation</td>
</tr>
<tr>
<td>5</td>
<td>2.7, 2.8</td>
<td>Related rates, differentials</td>
</tr>
<tr>
<td>6</td>
<td>3.1 – 3.3</td>
<td>Exponential, logarithmic functions and their derivatives</td>
</tr>
<tr>
<td>7</td>
<td>3.5, (3.6), 3.7</td>
<td>Inverse trig functions, (hyperbolic functions), L'Hospital's Rule</td>
</tr>
<tr>
<td>8</td>
<td>4.1, 4.2, 4.3</td>
<td>Maximum and minimum values, Mean Value Theorem, first derivative test</td>
</tr>
<tr>
<td>9</td>
<td>4.3, 4.4</td>
<td>Second derivative test, concavity and inflection points, curve sketching</td>
</tr>
<tr>
<td>10</td>
<td>4.5, 4.6</td>
<td>Optimization problems, Newton's method</td>
</tr>
<tr>
<td>11</td>
<td>4.7, 5.1, 5.2</td>
<td>Antiderivatives, area, definite integral and its properties</td>
</tr>
<tr>
<td>12</td>
<td>5.3, 5.4</td>
<td>Fundamental Theorem of Calculus and its applications, average value of a function</td>
</tr>
<tr>
<td>13</td>
<td>5.5, 7.1</td>
<td>Substitution rule, areas between curves</td>
</tr>
</tbody>
</table>

The text has some sections that contain both essential and nonessential topics. In order to ensure that essential topics receive full coverage, an instructor may omit the following topics.

1. Precise definitions of limits (pp. 31, 32, 64-67 of Sections 1.3, 1.6).
2. Linear approximation using differentials (pp. 135-136 of Section 2.8).

Note: Mathematics Department Policy requires that a comprehensive Final Examination be given in this course.

If any questions arise concerning this course, contact the Chair of the Calculus Committee.

5/14/12
CALCULUS I REDESIGNED

Who takes Math 122? Students enrolled in MATH 122 are typically first- and second-year students majoring in science or engineering; there is also a smattering of students intending to major in mathematics, undeclared students, and students from other colleges. Despite having met the prerequisites for the course, the students in our experience have gaps in their understanding of precalculus mathematics (e.g. trigonometry, functions, exponents, logarithms). Difficulties with these topics compound in the face of new material in MATH 122.

MATH 122 (Calculus I) is the first course and hence the cornerstone of the calculus sequence. This is a course with a traditionally low pass rate, not only on CSULB campus but across the CSU system (and indeed beyond). It is also one of the six "High Enrollment-Low Success" bottleneck courses in mathematics identified across the entire CSU system. This link provides information in this regard.

The redesign involves a number of coordinated components. Central among these is a suite of 75-minute tutorials, which students identified as "at-risk," either through low grades in the prerequisite coursework or low placement exam scores, are expected to attend in small groups once a week. The purpose of these tutorials is remediation. The tutorials will concentrate on the types of problem that were the most difficult to the students in the previous week's common online homework sets. Based on advice from experienced calculus instructors, the redesign team carefully chose the most fundamental problems and set up the homework assignments across all participating sections. The online homework software provides instant feedback to student solutions and allows student multiple attempts in solving problems, as well as additional practice problems and online tutorials. Other features include a sequence of three online milestone exams to be given before each midterm to provide pre-exam feedback to students and which can also serve to identify gaps and difficulties in students' understanding of major calculus concepts and procedures and to shunt students as needed into the tutorials.

WHY IN-HOUSE SEPPEMENTAL ACTIVITY (S.A.) SESSIONS?

Due to the difficulty level of Math 122, students need to spend a huge amount of time working on the problems and concepts in order to pass the course and build a knowledge base solid enough for success in subsequent courses. For many students, the built in activity sessions along with their personal study time are sufficient. The students we have identified as at-risk typically need additional tutorial assistance, which is the purpose of these in house S.A.’s.

Up until Fall 2014, Math 122 students could enroll in and attend a Supplemental Instruction (S.I.) section, through the Learning Assistance Center at the Horn Center, taught by undergraduate math majors. These were excellent resources that were optional for all students (although most attendees are the better students). The drawbacks of S.I.
were that a student was required to attend for the whole semester (if you did not register at the beginning of the semester, you couldn’t get in later), and there was not necessarily coordination with faculty teaching Math 122. The S.A. also created to provide an avenue where department faculty could provide direct input to the teaching assistants. In addition, students will be required to attend based on “at-risk” status, which may change dynamically over the course of the semester.

 WHO WILL ATTEND MY SESSION? DEFINITION OF AT-RISK STUDENTS

Consider the topics listed in the Math 111 and Math 113 course outlines. Imagine having a poor understanding of several of these topics. What would be your prognosis for passing Math 122? Over the years, student performance in these courses is a pretty good indicator for whether or not a student is at risk for getting a D, F or W in Math 122.

<table>
<thead>
<tr>
<th>MATH 111 GRADE</th>
<th>MATH 113 GRADE</th>
<th>AT RISK?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class not Necessary</td>
<td>Class non necessary</td>
<td>No</td>
</tr>
<tr>
<td>A or B</td>
<td>A or B</td>
<td>No</td>
</tr>
<tr>
<td>A or B</td>
<td>C</td>
<td>Yes</td>
</tr>
<tr>
<td>C</td>
<td>A or B</td>
<td>Yes</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>Yes</td>
</tr>
<tr>
<td>DFW</td>
<td>Any</td>
<td>Not Eligible</td>
</tr>
<tr>
<td>Any</td>
<td>DFW</td>
<td>Not Eligible</td>
</tr>
</tbody>
</table>

When students do not have grades for 111 or 113, GPA is used to help determine risk. These categories determine student risk status at the beginning of the semester. Students initially at risk stay at risk throughout the semester. Over the course of the semester, however, a student who gets a C or below on a midterm exam will be reassigned to the at risk group. Any student that has not taken any mathematics class within the prior academic year will be required to attend S.A.

In addition to categorically at-risk students, all freshmen should be encouraged to attend S.A. if they choose to.

 SAMPLE SYLLABUS

 This link provides logistical information for each component of the course. As you can see, there are many moving parts to the redesigned Math 122:

- Exams
- Quizzes
- Off-line Homework
- On-line Homework (WebAssign)
- On-line Benchmark Tests (via WebAssign)
- Grade Maintenance and Improvement/Supplemental Activity attendance
RUNNING THE SESSIONS

LOGISTICS
All Math 122 Supplemental Activity sessions will run in a location TBD. The following is the schedule of these sessions:

<table>
<thead>
<tr>
<th>Teaching Assistant</th>
<th>Day</th>
<th>Time</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diana Gonzales</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Genesis Islas</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The students in your S.A. may be from any participating section of Math 122. They will attend based on their schedule/availability. It is the responsibility of course instructors to notify students of their requirement to attend S.A.

In addition, if whiteboard markers are not available in the conference room, you may obtain them from the department office.

It is important that you show up on time to set up, and to answer any preliminary questions students may have.

YOUR RESPONSIBILITIES

1. Make sure you have received the worksheet for the week, and have gone through solutions to the problems before your first S.A. session.

2. Keep an attendance sheet (Excel spreadsheet).

3. Have a game plan for student engagement before each activity. Will students work in groups? Will they work individually? Will each student work on the board? You can speak with the members of the redesign team at any time for ideas on this.

4. Facilitate, don’t teach (see below), ensuring that EVERY student is involved to a reasonable degree.

5. It is not your responsibility to make sure students attend; you are responsible to provide an environment where students can maximize their contact time and quality of engagement with the material, under your supervision.
TEACHING STYLES: LECTURER VS FACILITATOR

<table>
<thead>
<tr>
<th>Lecturer</th>
<th>Facilitator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Takes student questions and gives them the step by step solutions</td>
<td>Uses student questions as starting points for a discussion in which students generate step by step solutions</td>
</tr>
<tr>
<td>Needs the writing instrument in his hand to be effective</td>
<td>Needs the writing instrument in the students’ hands to be effective</td>
</tr>
<tr>
<td>Is expected to know the destination, and be prepared to explain every possible road to get there</td>
<td>Is expected to know the destination, and be prepared to help students discover the most effective road(s) for them to get there</td>
</tr>
<tr>
<td>Speaks to students as one large group, whose learning progress is gauged by the responses of only the most vocal</td>
<td>Listens to students as individuals, and learning progress is gauged by the performance of each individual or small group</td>
</tr>
<tr>
<td>Values whether a student is right or wrong more than their participation</td>
<td>Values student attempts at solutions and/or participation more than whether students are right or wrong.</td>
</tr>
<tr>
<td>Speaks more than he listens</td>
<td>Listens more than he speaks</td>
</tr>
<tr>
<td>States what he knows about the topic of the day, and expects students to provide what they need to develop competence.</td>
<td>First determines what students know about the topic of the day, and expects himself to provide what they need in order to develop competence.</td>
</tr>
</tbody>
</table>

The following [link](#) provides a more general perspective on the differences and similarities between lecturing (teaching) and facilitating.

ROLE PLAYING
Work through some scenarios with the S.A. leaders, where faculty plays the student. Give them advice on how to improve facilitation.

USING TECHNOLOGY IN SUPPLEMENTAL ACTIVITY
A critical learning outcome for this course is that students understand how to translate between verbal, symbolic, numerical, and graphical representations of functions. For many fundamental functions the use of technology is not necessary, and may indeed be a crutch for students. Is it really necessary to use a calculator to calculate the exponent in the derivative of \( y = x^{2/5} \)? Do we really need Wolfram Alpha to graph \( f(x) = x^2 - x \)? While we want to help students build confidence in certain fundamental tasks, as the semester forges on, they will encounter a multitude of functions that are too intricate to be graphed from a simple table of \( x \) and \( y \) values. Thus it is critical that they use technology to aide their ability to associate graphical displays with symbolic expressions, and recognize
patterns in these associations. You are encouraged to be handy with graphing calculators, and should meet with any of the 122 Redesign team if you are rusty.

As critical as technology is for understanding calculus I, the use of laptops is not allowed in S.A. sessions. Students may attempt to use the S.A. time as an opportunity to get caught up on their WebAssign homework, and you must ensure that they understand that is not the appropriate time. If they have questions on WebAssign, recommend that they attend their professor’s office hours. **There should be no exceptions to this rule.**

**FREQUENTLY ASKED QUESTIONS**
<table>
<thead>
<tr>
<th>WEEK</th>
<th>MONDAY</th>
<th>TUESDAY</th>
<th>WEDNESDAY</th>
<th>THURSDAY</th>
<th>FRIDAY-SUNDAY?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Jan 20</td>
<td>SECTION 1.1,2</td>
<td>QUIZ 1</td>
<td>1.3,4</td>
<td>WA: Beginning of Course Survey</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
<td>WA Set 0 (1.1,2) 1.5</td>
<td>QUIZ 2</td>
<td>1.6</td>
<td>WA Set 1 (1.3,4)</td>
</tr>
<tr>
<td>3</td>
<td>Feb 3</td>
<td>2.1,2.2</td>
<td>QUIZ 3</td>
<td>2.2-2.4</td>
<td>WA Set 2 (1.5,6)</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>2.4</td>
<td>Benchmark 1</td>
<td>2.5</td>
<td>WA Set 3 (2.1,2.2,2.3)</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>WA Set 4a (2.4) 2.6,2.7</td>
<td>QUIZ 4</td>
<td>MIDTERM 1 (1.1-2.4)</td>
<td>WA Set 4b (2.5,2.6)</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>2.7,2.8</td>
<td>QUIZ 5</td>
<td>3.1,3.3b</td>
<td>WA Set 5 (2.7,2.8)</td>
</tr>
<tr>
<td>7</td>
<td>Mar 2</td>
<td>3.2,3.3a</td>
<td>QUIZ 6</td>
<td>3.5</td>
<td>WA Set 6 (3.1-3.3)</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>3.7</td>
<td>Benchmark 2</td>
<td>4.1</td>
<td>WA Set 7 (3.5-3.7)</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
<td>WA Set 8a (4.1) 4.2</td>
<td>QUIZ 7</td>
<td>MIDTERM 2 (2.5-4.1)</td>
<td>WA Set 8b (4.2)</td>
</tr>
<tr>
<td>10</td>
<td>23</td>
<td>4.3,4.4</td>
<td>QUIZ 8</td>
<td>4.3,4.4</td>
<td>WA Set 9 (4.3,4.4)</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SPRING BREAK</td>
</tr>
<tr>
<td>12</td>
<td>Apr 6</td>
<td>4.5</td>
<td>QUIZ 9</td>
<td>4.6,4.7</td>
<td>WA Set 10 (4.5,4.6)</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>4.7,5.1</td>
<td>Benchmark 3</td>
<td>5.2,5.3</td>
<td>WA Set 11 (4.7,5.1,5.2)</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>5.4</td>
<td>QUIZ 10</td>
<td>MIDTERM 3 (4.2-5.3)</td>
<td>WA Set 12 (5.3,5.4)</td>
</tr>
<tr>
<td>15</td>
<td>27</td>
<td>5.5</td>
<td></td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>May 4</td>
<td>7.1</td>
<td>QUIZ 11</td>
<td>REVIEW</td>
<td>WA Set 13 (5.5,7.1)</td>
</tr>
<tr>
<td>17</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td>FINALS WEEK</td>
</tr>
</tbody>
</table>
## WEEKLY S.A. ASSIGNMENTS AND SOLUTIONS

<table>
<thead>
<tr>
<th>WEEK</th>
<th>SECTIONS</th>
<th>TOPICS</th>
<th>LEARNING OBJECTIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1,1.2</td>
<td>Essential Functions, Algebra Review</td>
<td>Students will recall major classifications of functions; Students will review algebraic techniques useful for calculating limits (Factoring, Expanding, Rationalizing, Combining fractions-FERC)</td>
</tr>
<tr>
<td>2</td>
<td>1.4</td>
<td>Limits</td>
<td>Students will learn how to categorize limit problems, and identify which category a given problem falls into (FERC)</td>
</tr>
<tr>
<td>3</td>
<td>1.5,1.6</td>
<td>Continuity, Infinity Limits</td>
<td>Students learn to determine continuity. Students develop strategies for identifying and solving infinity limits.</td>
</tr>
<tr>
<td>4</td>
<td>2.1-2.3</td>
<td>Intro to Derivatives/ Midterm 1 Review</td>
<td>Students practice context-lacking problems to develop a feel for the mechanics of derivative calculation, both by definition, and by formula.</td>
</tr>
<tr>
<td>5</td>
<td>2.4-2.6</td>
<td>Differentiation Rules</td>
<td>Students identify which rule(s) is (are) applicable for calculating the derivative of a function, and which is the shortest path to the solution</td>
</tr>
<tr>
<td>6</td>
<td>2.7</td>
<td>Related Rates/Review</td>
<td>Students explore the steps to building basic mathematical models. Students review past topics they struggled with most (based on responses to Google survey).</td>
</tr>
<tr>
<td>7</td>
<td>3.1-3.3</td>
<td>Exponential and Log Functions</td>
<td>Students strengthen their grasp on fundamental properties of exponential and logarithmic functions, and address common misconceptions</td>
</tr>
<tr>
<td>8</td>
<td>3.5,3.7</td>
<td>Inverse Trig and L’Hospital</td>
<td>Students strengthen their grasp on inverse functions, and discover fundamental properties of inverse trig functions. Students learn how to identify and solve problems where l’Hospital’s rule is applicable, within the greater context of finding limits discussed prior.</td>
</tr>
<tr>
<td>9</td>
<td>2.4-3.7</td>
<td>Midterm 2 Review</td>
<td>Students review past topics they struggled with most (based on responses to Google survey).</td>
</tr>
<tr>
<td>10</td>
<td>4.4</td>
<td>Curve Sketching</td>
<td>Students gain confidence identifying key function characteristics from symbolic expression of a function, and using graphing calculators to verify findings.</td>
</tr>
<tr>
<td>11</td>
<td>4.5</td>
<td>Optimization</td>
<td>Students explore the steps to building basic mathematical models.</td>
</tr>
<tr>
<td>12</td>
<td>4.7,5.1-3</td>
<td>Understanding anti-differentiation</td>
<td>Students explore and connect graphical, numerical and symbolic notions of antiderivative.</td>
</tr>
<tr>
<td>13</td>
<td>5.4</td>
<td>The FTC/Midterm 3 Review</td>
<td>Students discover the meaning of the FTC. Students review past topics they struggled with most (based on responses to Google survey).</td>
</tr>
<tr>
<td>14</td>
<td>1.1-5.4,5.5</td>
<td>Review, Basic Substitution</td>
<td>Students review past topics they struggled with most (based on S.A. instructor’s observation).</td>
</tr>
<tr>
<td>15</td>
<td>5.5</td>
<td>Substitution and Final Review</td>
<td>Students develop strategies for determining expressions for “u” when using the substitution technique. Student’s make connections with the chain rule.</td>
</tr>
</tbody>
</table>
TOPICS: Representing Functions, Finding Domain, Calculating Limits, Determining Continuity

### 1.1 - FUNCTIONS AND THEIR REPRESENTATIONS

There are FOUR representations of functions. What are they?

1. 
2. 
3. 
4. 

This can be found on page______ of the textbook.

### 1.2 - ESSENTIAL FUNCTIONS

1. (see pg. 15) A rational function \( f \) is the ratio of two____________________: \( f(x) = \) __________

   Thus its domain is \( \{ x | \text{________________} \} \).

   EXAMPLE: \( f(x) = \)

   DOMAIN:

2. A root function \( g \) takes the form \( g(x) = \)

   Thus its domain is \( \{ x | \text{________________} \} \).

   EXAMPLE:

   DOMAIN:

Exercises: For each function, find the domain, plot the graph
1.3-THE LIMIT OF A FUNCTION

In this class, when we say \( \lim_{x \to a} f(x) \) does not exist (DNE), what does that mean? In other words, what needs to happen at or near \( a \) in order for the answer to be DNE?

1.4-CALCULATING LIMITS

1. State the DSP (see pg 37)

2. When the DSP leads to 0/0, we use ______________. Then, when we do this, what step will occur regardless of the type of function? Why?

3. How many types of limits are there? What are they?

4. Each limit problem in which the DSP leads to 0/0 will be solved by using one of the following algebra techniques:

<table>
<thead>
<tr>
<th>Types of Functions</th>
<th>Techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
</tbody>
</table>

Exercises: For each function find the indicated limit (same functions as above)
WARM-UP PROBLEMS
1. Factor $x^3 - 64$.

2. Solve $-(x - 2)^{-3/2} + (x - 2)^{1/2} = 0$.

3. Find the equation (in point-slope form) of the line parallel to $2x - 3y = 5$ and through the point ($-7, 1$).

1.5 CONTINUITY
1. There are ________ types of discontinuities. They are:

2. Graph $f(x)$ if $\lim_{x \to 3} f(x) = 4$, $\lim_{x \to 5} f(x) = -\infty$, $\lim_{x \to -4} f(x) = 2$, $f(3) = 1$.

The gravitational force exerted by the earth on a unit mass at a distance $r$ from the center of the planet is

$$F(r) = \begin{cases} \frac{GM}{r^3} & \text{if } r < R \\ \frac{GM}{r^2} & \text{if } r \geq R \end{cases}$$

where $M$ is the mass of Earth, $R$ is its radius, and $G$ is the gravitational constant. Is $F$ a continuous function of $r$?
4. For what value of the constant $c$ is the function $f$ continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 4x & \text{if } x < 3 \\ x^3 - cx & \text{if } x \geq 3 \end{cases}$$

5. Consider the following equation.

$$\cos x = x^3$$

(a) Prove that the equation has at least one real root.

(b) Use your calculator to find an interval of length 0.01 that contains a root. (Enter your answer using interval notation.)
1.6 LIMITS INVOLVING INFINITY

6. Use limits to find the horizontal and vertical asymptotes of \( f(x) = \frac{3x^2 - 6x}{x^2 - 3x + 2} \). Your final answer should use limits to describe the behavior of the function near the asymptotes, and you must show work for each (See Eg.1 on pg. 58 and Eg.5 on pg. 62). For example,

HA is __________, since ______________________________

Also, state where the function is continuous. Use interval notation.
Sketch the curve

7. The function \( f(x) \) is an odd function and \( \lim_{x \to \infty} f(x) = 32 \), then
\[
\lim_{x \to -\infty} f(x) = \boxed{\text{__________}}
\]
8. Give a formula for a function with infinite discontinuity at \( x = -2 \), removable discontinuity at \( x = 3 \), and horizontal asymptote at 1. Sketch.

\[ f(x) = \]

9.

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

\[
\lim_{x \to \infty} \frac{\sqrt{9x^2 - x}}{x^3 + 6}
\]

10.

Evaluate the limit

\[
\lim_{x \to \infty} \left( \sqrt{x^2 + 3} - \sqrt{x^2 - 11} \right)
\]
WARM-UP PROBLEMS

1. Simplify \( 5x^{4/5}(3x^2 - 7x + 2x^{-3}) + \frac{1}{2}x^{-1/5}(-4x^2 + x^3 - 8x^{-2}) \)

2. Find the domain of \( r(t) = \sqrt{t^4 - 16} \)

2.1 DERIVATIVES AND RATES OF CHANGE

The graph of \( f(x) \) is shown above.

1. Find the average rate of change between \( x=2 \) and \( x=8 \) (show your formula and your work).

2. On the graph above, draw the line that connects the points on the curve where \( x=2 \) and \( x=8 \). This line is called a __________________ line. The slope of this line is ________________.

3. Do you think there is a location between \( x=2 \) and \( x=8 \) where the slope of the tangent line is equal to your answer to #1? Circle your answer: YES NO. The approximate \( x \)-value where this happens is \( x= \_________. \)

2.2 THE DERIVATIVE AS A FUNCTION

1. State the approximate locations where \( f'(x)=0: \) _______ _______ _______ _______

2. State the approximate intervals where \( f'(x)>0: \) _______ _______ _______ _______

3. State the approximate intervals where \( f'(x)<0: \) _______ _______ _______ _______

4. Sketch a rough graph of \( y=f'(x) \) on the figure above.
### 2.3 BASIC DIFFERENTIATION FORMULAS and 2.4 PRODUCT AND QUOTIENT RULES

Find the derivative of the function using multiple methods. Indicate the method you used where blank, and show all answers are the same (brainstorm on the board before you write your final answers):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $f(x) = 5x^3$</td>
<td>METHOD #2: Definition of the derivative</td>
<td>METHOD #1: ____________________________</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. $g(w) = -\frac{7}{w^3}$</td>
<td>METHOD #2: ____________________________</td>
<td>METHOD #3: Definition of the derivative</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. $h(x) = \frac{2x^2 + 6x + 4}{\sqrt{x}}$</td>
<td>METHOD #2: ____________________________</td>
<td>METHOD #1: ____________________________</td>
</tr>
</tbody>
</table>
A particle moves according to a law of motion \( s = f(t), \ t \geq 0, \) where \( t \) is measured in seconds and \( s \) in feet.

\[
f(t) = t^3 - 12t^2 + 36t
\]

(a) Find the velocity at time \( t \).

\[
v(t) =
\]

(b) What is the velocity after 4 s?

\[
v(4) = \underline{\text{ }} \text{ ft/s}
\]

(c) When is the particle at rest? (Enter your answer as a comma-separated list.)

\[
t = \underline{\text{ }}
\]

(d) When is the particle moving in the positive direction? (Enter your answer in interval notation.)

\[
t = \underline{\text{ }}
\]

(e) Find the total distance traveled during the first 8 s.

\[
\underline{\text{ }} \text{ ft}
\]

(f) Draw a diagram to illustrate the motion of the particle.
(g) Find the acceleration at time $t$.

\[ a(t) = \]

Find the acceleration after 8 s.

\[ a(8) = \boxed{\text{ft/s}^2} \]

(h) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 8$. 

\[ s \]

\[ v \]

\[ a \]

\[ s \]

\[ v \]

\[ a \]

\[ s \]

\[ v \]

\[ a \]
WARM UP PROBLEMS

1. Expand \((x-5y)^3-(-2x+y)^3\)

2. Fill in the missing spaces

| \(0 \leq x \leq \frac{\pi}{2}\) | \(\pi/4\) |  \\
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\cos x)</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>(\tan x)</td>
<td></td>
<td>(\sqrt{3})</td>
</tr>
</tbody>
</table>

MIDTERM 1 REVIEW PROBLEMS

1. Prove that the equation has at least one real root: \(x^5 - 3x^2 + 5x = 10\)

2. Evaluate the limit, if it exists. You must show work to get credit. Where is the function continuous? State the types and locations of all discontinuities. Sketch the graph to see if your answer makes sense.

a. \(\lim_{x \to -3} \frac{x - 2}{x^2 + x - 6}\)

b. \(\lim_{x \to \infty} \frac{x - 2}{x^2 + x - 6}\)

d. \(\lim_{t \to \infty} \frac{\sqrt{t} + 3t^2}{2t - t^2}\)
3. Differentiate the function. State which rule you are using, show your steps, and simplify.
a. \( R(x) = (2x^5 - 7)^2 \) (No chain)

\[ b. \quad Q(x) = \frac{\sqrt[3]{x} - 1}{\sqrt[3]{x} + 1} \]

c. \( g(x) = \frac{x^3 - 2x^2 \sqrt{x^2}}{x^{3/2}} \).

d. \( f(x) = (12x^2 - 7x^4)(x^8 - 3) \). Do in two ways and compare answers.

3. Find the equation of the tangent line to \( h(x) = 3x^7 \sec x \) at \( x = \pi \).

Now choose a problem from a previous S.A. Assignment to explain to your group.
PROBLEM: S.A. HW#___________ PROBLEM#___________.
WARM UP PROBLEMS

1. Find three representations of y in terms of x, z, and \( \theta \).

\( y = \) ____________________________
\( y = \) ____________________________
\( y = \) ____________________________

2. State the properties of exponential and logarithmic functions; if no property applies write “NONE”

<table>
<thead>
<tr>
<th>EXPONENTIAL</th>
<th>LOGARITHMIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXPRESSION</td>
<td>EQUALS</td>
</tr>
<tr>
<td>( e^x e^y )</td>
<td>( e^{x-y} )</td>
</tr>
<tr>
<td>( e^{xy} )</td>
<td>( e^y )</td>
</tr>
</tbody>
</table>

REVIEW PROBLEMS

1. State the general rules of differentiation


2. Find the derivative using the product, then the quotient rules: \( R(x) = \frac{3x}{\sqrt{x^2 - 1}} \), and show equal.

Product

Quotient
2.7 RELATED RATES

1. At noon, ship A is 170 km west of ship B. Ship A is sailing east at 40 km/h and ship B is sailing north at 15 km/h. How fast is the distance between the ships changing at 4:00 PM?

2. Two sides of a triangle are 6 m and 12 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is \( \frac{\pi}{3} \) rad.

3. If two resistors with resistances \( R_1 \) and \( R_2 \) are connected in parallel, as in the figure below, then the total resistance \( R \), measured in ohms (\( \Omega \)), is given by

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.
\]

If \( R_1 \) and \( R_2 \) are increasing at rates of 0.3 \( \Omega \)/s and 0.2 \( \Omega \)/s, respectively, how fast is \( R \) changing when \( R_1 = 60 \) \( \Omega \) and \( R_2 = 100 \) \( \Omega \)? (Round your answer to three decimal places.)
MATH 122:: SUPPLEMENTAL ACTIVITY HW # 6

WARM UP PROBLEMS

1. Solve for $x$: $\log_2 96 = 3 + \log_2 x$ (no calculator)

2. Simplify: $3\log(x^2 - 25) - 2\log(2x^3 + 13x^2 + 15x) + \log(14x + 21)$

3. Find the domain and inverse of the function $f(x)=3 + \ln(7+x)$.

4. WHITEBOARD PROBLEM: Find the $x$-values where the slope of the tangent to the following curves is equal to zero. Sketch the graph on the board. What seems to be true about the curve at these points?
   A. $f(x) = (x^3 - 12x)^2$
   B. $g(x) = \frac{x^2}{x - 2}$

5. Fill out the table without using a calculator. Identify the angles in the $2^{nd}$, $3^{rd}$, and $4^{th}$ quadrants that correspond to $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ and state the sine, cosine and tangent of these values.

<table>
<thead>
<tr>
<th>$\theta$(Deg)</th>
<th>0</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$(Rad)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sin\theta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\cos\theta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tan\theta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $\theta$(Deg) |      |      |      |      |      |
| $\theta$(Rad) |      |      |      |      |      |
| $\sin\theta$  |      |      |      |      |      |
| $\cos\theta$  |      |      |      |      |      |
| $\tan\theta$  |      |      |      |      |      |
2.6 IMPLICIT DIFFERENTIATION
1. Use implicit differentiation to find an equation of the tangent line to the curve at the given point
A. \( y \sin 12x = x \cos 2y, \ (\pi / 2, \pi / 4) \).
B. \( x^2 - y^2 = (3x^2y - y^4)^3, \ (0, -1) \).

3.1-3.3-EXPONENTIAL AND LOGARITHMIC FUNCTIONS
1. Describe the end behavior of the natural exponential and logarithmic functions with limits. Determine the x- and y- intercepts by SOLVING the appropriate equations. Illustrate with a sketch.

End behavior of \( e^x \)

LEFT:

RIGHT:

x-intercept equation:

y-intercept equation:

End behavior of \( \ln x \)

LEFT:

RIGHT:

x-intercept equation:

y-intercept equation:

2. Find the derivative
A. \( y = \frac{4x^2e^{2x}}{5} \)
B. \( g(x) = \frac{8}{3e^{2x}} \)
C. \( M(x) = 6e^{\cos x} \)
### Warm Up Problems
Sketch \( f(x) \) and \( f'(x) \) in different colors on the same graph on the board. Then answer the following questions about \( f(x) \).

i. Find the domain of \( f(x) \) (interval notation).
ii. Find where \( f(x) \) has a maximum or minimum value.
iii. Find where \( f(x) \) is increasing, and where it is decreasing (interval notation).

A. \( f(x) = \frac{x^2}{x^2 - 4} \)  
B. \( f(x) = x^2 e^{-4x^2} \)

1. The diameter of a sphere is increasing at a rate of 5mm/s. How fast is the volume increasing when the radius is 30mm?

2. Evaluate the limit

   a. \( \lim_{x \to 3} \frac{x^2 - 7x + 12}{4x^2 - 8x - 12} \)
   b. \( \lim_{x \to \infty} \frac{3x}{\sqrt{2 + 5x^2}} \)
   c. \( \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} \)

### 3.3 Derivatives of Exponential and Logarithmic Functions
For #s 1&2, find the domain, and all asymptotes. Then find the derivative as indicated.

1. \( h(x) = \ln(x^2 - 7x + 12) \)

<table>
<thead>
<tr>
<th>Domain:</th>
<th>HA:</th>
<th>VA:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h'(x) ) METHOD 1</td>
<td>( h'(x) ) METHOD 2</td>
<td></td>
</tr>
</tbody>
</table>

2. \( r(x) = e^{2x-6} \)

<table>
<thead>
<tr>
<th>Domain:</th>
<th>HA:</th>
<th>VA:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r'(x) ) METHOD 1</td>
<td>( r'(x) ) METHOD 2</td>
<td></td>
</tr>
</tbody>
</table>
3. Find the derivative of $y=x^2$.

### 3.5 INVERSE TRIGONOMETRIC FUNCTIONS
For the following, find the domain of the function and of its derivative. Then evaluate the limit as $x$ approaches $0$ for both.

<table>
<thead>
<tr>
<th>A. $f(x) = \sin^{-1}(5x-1)$</th>
<th>B. $g(x) = \tan^{-1}(x^{3/2})$</th>
</tr>
</thead>
</table>
1. Consider the equation $9x^2 + y^2 = 9$. (A) Find the equation for the top half of this curve, and plot it.

\[ y = \sqrt{9 - 9x^2} \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

(B) Find $y'$ using implicit differentiation:

2. Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 2 m/s, how fast is the area of the spill increasing when the radius in 30m?

(D) Show your answers in (B) and (C) are equal.
3. (A) Find the linearization \( L(x) \) of the function \( f(x) = \sqrt{2-x} \) at \( a \).

(B) Use your answer to (A) to approximate \( \sqrt{1.97} \) and \( \sqrt{2.1} \).

4. Let \( P(x) = \frac{-9e^x}{1-3e^x} \).  
(A) Find the domain of \( P(x) \) (show work).

(B) Identify the asymptotes of this function (HINT: Look at the graph before answering). **Use limit notation** to justify your answer

<table>
<thead>
<tr>
<th>HA</th>
<th>VA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(C) From the graph, does the function appear to be one-to-one? **YES**  **NO**
Use algebra to find the value of \( x \) for which \( P(x)=4 \).

(D) Find \( P'(x) \)
5. Which is faster, exponential or polynomial growth? Compare the end behavior of an exponential

6. Use two methods to find the following derivatives
   (A) \( h(x) = \ln(\sin^2 x) \)

   I. 
   II. 

   (B) \( r(x) = \frac{1}{\ln x} \)

   I. 
   II. 

7. Find the limit
   (A) \( \lim_{t \to 0} \frac{e^{2t} - 1}{\sin t} \)

   (B) \( \lim_{x \to 0} x^{\sqrt{x}} \)
8. (A) Derive (prove) the formula for the derivative of $\tan^{-1}x$.

(B) Let $G(x) = \tan^{-1}\sqrt{1+3x}$. Find the derivative of $G(x)$.

(C) Find the domain of $G(x)$ and of its derivative.

(D) Find the limit as $x \to \infty$ for $G(x)$ and its derivative.
For each function

1] State the domain
2] Find the horizontal and vertical asymptotes (if any)
3] Find the first derivative
4] Find the values of \( x \) where the first derivative is zero or DNE (critical numbers)

5] Find the second derivative
6] Find the values of \( x \) where the second derivative is zero or DNE (inflection points)
7] Sketch the graph with your calculator. Can you see any relationship between your answers to 1-6 and the graph?

A. \( f(x) = 10x^3 + 30x^2 - 90x \)

B. \( f(r) = r^3 + r \)
C. \( g(x) = 3\sin x - \sin^3 x \)

D. \( v(x) = \frac{x}{x^2 - 9} \)
E. \( s(t) = \frac{x^2}{x^3 - 3} \)

F. \( h(x) = x \ln x \)
G. $a(t) = e^{2t} - e^{t}$
MATH 122:: SUPPLEMENTAL ACTIVITY HW # 10

WARM-UP PROBLEMS

1. Fill in the blank boxes

<table>
<thead>
<tr>
<th>FUNCTION $f(x)$</th>
<th>DERIVATIVE $f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12x^5 - 3\cos^2 x^4$</td>
<td>$9x^6$</td>
</tr>
<tr>
<td>$\frac{\sin^{-1} x}{\cos^{-1} x}$</td>
<td>$\frac{1}{x} + \frac{1}{1 + x^2} + \frac{1}{e^x}$</td>
</tr>
<tr>
<td>$\sin^{-1} x$</td>
<td>$5x^3 + 7\cos x - 2$</td>
</tr>
</tbody>
</table>

4.3-4.4 CURVE SKETCHING

For each function

A] State the domain
B] Find x- and y- intercepts
C] Identify symmetry
D] Find the horizontal and vertical asymptotes (if any)
E] Identify intervals of increase/decrease
F] Identify local min and max values

G] Identify intervals of concavity and inflection points
H] Sketch the curve

1. $v(x) = \frac{x^2}{x^2 + 9}$

Let’s prepare by finding $v'(x)$ and $v''(x)$. MAKE SURE YOUR ANSWER MATCHES WITH A CLASSMATE BEFORE YOU PROCEED. Pair up and take turns explaining to the teaching assistant how you found your derivative on the board (one person show $v'(x)$, other person $v''(x)$). Be sure to simplify your answer.

$v'(x) =$

$v''(x) =$
1. \( v(x) = \frac{x^2}{x^2 + 9} \)

<table>
<thead>
<tr>
<th>A. DOMAIN</th>
<th>C. SYMMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(x) ) is a rational function, so domain is where ( ) ( \neq 0 ):</td>
<td>Find ( v(-x) = )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D. HORIZ. ASYMPTOTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find the following limit:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. y-INTERCEPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Where ( v(x) ) crosses y-axis, so set ( ) ( = 0 ):</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x-INTERCEPT(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Where ( v(x) ) crosses x-axis, so set ( ) ( = 0 ):</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VERT. ASYMPTOTE(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check the following limit:</td>
</tr>
</tbody>
</table>
### E. INC/DEC

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$v'(x)$</td>
<td></td>
</tr>
<tr>
<td>$v(x)$</td>
<td></td>
</tr>
</tbody>
</table>

### F. LOCAL MAX/MIN

The function $v(x)$ has a local ______ value of ______________
that occurs at ___________

The function $v(x)$ has a local ______ value of ______________
that occurs at ___________

The function $v(x)$ has a local ______ value of ______________
that occurs at ___________

### SUMMARY: POINTS TO BE INCLUDED IN THE SKETCH

#### A. DOMAIN (interval notation)

#### B. INTERCEPTS

* y: (___, ___)
* x: (___, ___); (___, ___);
  (___, ___)

#### C. SYMMETRY

Odd? Even? Periodic? None?

#### D. ASYMPTOTES

H.A. Equation:

V.A. Equation(s):

#### E. INC/DEC

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$v''(x)$</td>
<td></td>
</tr>
<tr>
<td>$v(x)$</td>
<td></td>
</tr>
</tbody>
</table>

#### F. LOCAL EXTREMA

Min: (___, ___); (___, ___)

Max: (___, ___); (___, ___)

#### G. INFLECTION POINTS

(___, ___); (___, ___)
2. \( R(x) = -3x^5 + 5x^3 \)

\[ R'(x) = \]
\[ R''(x) = \]

C. SYMMETRY
Find \( R(-x) = \)

A. DOMAIN
\( R(x) \) is a ______________ function, so domain is

D. HORIZ. ASYMPTOTE
Find the following limit:

B. y-INTERCEPT
Where \( R(x) \) crosses y-axis, so set ______ =0:

x-INTERCEPT(s)
Where \( R(x) \) crosses x-axis, so set ______ =0:

VERT. ASYMPTOTE(s)
Check the following limit:
E. INC/DEC

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( R'(x) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R(x) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F. LOCAL MAX/MIN

The function \( R(x) \) has a local ______ value of ______________ that occurs at ____________

The function \( R(x) \) has a local ______ value of ______________ that occurs at ____________

The function \( R(x) \) has a local ______ value of ______________ that occurs at ____________

G. CONCAVITY

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( R''(x) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R(x) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SUMMARY: POINTS TO BE INCLUDED IN THE SKETCH

A. DOMAIN (interval notation)

B. INTERCEPTS
- y: (_____, ____)
- x: (_____, ____); (_____, ____); (_____, ____)

C. SYMMETRY
Odd? Even? Periodic? None?

D. ASYMPOTOTES
H.A. Equation:
\[ _____ = _____ \]
V.A. Equation(s):
\[ _____ = _____; _____ = _____ \]

F. LOCAL EXTREMA
Min: (_____, ____); (_____, ____)
Max: (_____, ____); (_____, ____)

G. INFLECTION POINTS
(_____, ____); (_____, ____)

MARY: POINTS TO BE INCLUDED IN THE SKETCH

A. DOMAIN (interval notation)

B. INTERCEPTS

C. SYMMETRY
Odd? Even? Periodic? None?

D. ASYMPOTOTES
H.A. Equation:
\[ _____ = _____ \]
V.A. Equation(s):
\[ _____ = _____; _____ = _____ \]

F. LOCAL EXTREMA
Min: (_____, ____); (_____, ____)
Max: (_____, ____); (_____, ____)

G. INFLECTION POINTS
(_____, ____); (_____, ____)

G. CONCAVITY

<table>
<thead>
<tr>
<th>INTERVAL</th>
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<tbody>
<tr>
<td>( R''(x) )</td>
<td></td>
<td></td>
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<tr>
<td>( R(x) )</td>
<td></td>
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</table>
4.5 OPTIMIZATION PROBLEMS

3. Consider the following problem: Find two numbers whose sum is 23 and whose product is a maximum.
(a) Make a table of values, like the following one, so that the sum of the numbers in the first two columns is always 23. On the basis of the evidence in your table, estimate the answer to the problem.

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

(b) Use calculus to solve the problem and compare with your answer to part (a). Use two methods to verify that the product is a maximum.

(c) Can you think of how this problem could be reworded as a geometry problem with perimeter and area of a rectangle? Rephrase the problem below.
WARM-UP PROBLEMS

1. At noon, ship A is 180 km west of ship B. Ship A is sailing east at 40 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

2. Simplify \(3x^2 - 5x^{2/7} - 75x^{5/5} - 30x^{-1/2}\)

4.5 OPTIMIZATION

1. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

2. A piece of wire 29 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle.
   (a) How much wire should be used for the square in order to maximize the total area?
   (b) How much wire should be used for the square in order to minimize the total area?

3. What are the steps in an optimization problem? Write down the steps you would recommend if you were teaching your friend how to do a 4.5 problem.

   STEP 1:
   STEP 2:
   STEP 3:
   STEP 4:
   STEP 5:

4.7 ANTIDERIVATIVES

Find \(f(x)\):

1. \(f''(x) = -2 + 12x - 12x^2, \quad f(0) = 9, \quad f'(0) = 16\)

2. \(f''(t) = 4e^t + 2 \sin t, \quad f(0) = 0, \quad f(\pi) = 0, \quad f''(0) = 7\)
1. Let \( f(x) = x^5 - 2x \).

A. Find the Domain and Asymptotes

B. Find x- and y- intercepts

C. Determine Symmetry

D. Determine intervals of increase and decrease, and local extrema. Put the information in the table below as shown in class. You may not need the entire table.

E. Determine intervals of concavity. Put the information in the table below as shown in class. You may not need the entire table.

F. Sketch the graph, based on the above. Plot intercepts, local extrema and inflection points.
2. Let \( f(x) = 3x \ln x \).

<table>
<thead>
<tr>
<th>A. Find the Domain and Asymptotes</th>
<th>B. Find x- and y-intercepts</th>
<th>C. Determine Symmetry</th>
</tr>
</thead>
</table>

D. Determine intervals of increase and decrease, and local extrema. Put the information in the table below as shown in class. You may not need the entire table.

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</table>

E. Determine intervals of concavity. Put the information in the table below as shown in class. You may not need the entire table.

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</tbody>
</table>

F. Sketch the graph, based on the above. Plot intercepts, local extrema and inflection points.
3. State the Mean Value Theorem (MVT). Does it hold for \( f(x) = 2x^3 - 7x^2 + x \) on \([1,3] \)?

4. A farmer with 1200ft of fencing wants to enclose a rectangular area and then divide it into four pens (four sections) with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

5. Use Newton’s method to find the third approximation of an x-intercept for the function in question 1, \( f(x) = x^5 - 2x \). Use \( x_1 = 1.3 \).
6. Find \( f(x) \), given the following information.

\[ a. \quad f'(x) = \frac{8x^4 - x^3 + 12x^2}{x^4} \]

\[ b. \quad f'(x) = 7x^{2/5} + 4x^{-4/5} - \frac{3}{1 + x^2} \]

\[ c. \quad f''(x) = x^2 + 2\sin x + \cos x, \quad f(0) = 5, \quad f'(0) = 4. \]

7. Consider the area under the curve \( y = 5 - x \) from \( x = -1 \) to \( x = 13 \).

\( a. \) Approximate the area with \( L_7, R_7 \) and \( M_7 \).

\( b. \) Express the exact area as a Riemann sum.

\( c. \) Express the area as a definite integral.

\( d. \) Find the exact area by drawing the graph and using geometry.
Consider \( f(x) = x - \frac{x^2}{2}, \ 0 \leq x \leq 4 \). Plot this equation precisely on the axes provided below.

1. Find \( L_4 \), drawing rectangles in \textit{pencil}.  
2. Find \( M_4 \), drawing rectangles in \textit{pen}.

Now find the exact area using antiderivatives. What is your error in each case?

\[
\text{Error (L}_4\text{)} = \text{Error (M}_4\text{)} = \]
3. Find the following antiderivatives, stating the rule you are using

\[
\begin{array}{ccc}
\int x^2 \, dx & \int \frac{1}{x^2} \, dx & \int (x^2)^3 \, dx \\
\text{RULE:} & \text{RULE:} & \text{RULE:}
\end{array}
\]

4. Find the following antiderivatives, stating the rule you are using

\[
\begin{array}{ccc}
\int x^2 + 1 \, dx & \int \frac{1}{x^2 + 1} \, dx & \int (x^2 + 1)^3 \, dx \\
\text{RULE:} & \text{RULE:} & \text{RULE:}
\end{array}
\]

5. Consider \( \int \frac{1}{x^4 + 1} \, dx \). Can you find the antiderivative of this function? **YES NO**.

If YES, find it, and take the derivative of your answer to verify. If NO, make a minor change to the function by multiplying it by a power function. Verify your antiderivative to the new function is correct.
### 1. Let \( f(x) = \frac{x^2 - 7x}{x^2 - 6x - 7} \).

<table>
<thead>
<tr>
<th>A. State all asymptotes of this function</th>
<th>B. List the locations and types of discontinuities</th>
<th>C. Find ( \lim_{x \to 7} f(x) ) in two ways:</th>
</tr>
</thead>
</table>

### 3. Find the derivative of \( f(x) = \frac{1}{x^2} \) in three different ways (state the rule you are using):

| A. Definition of derivative | B. Rule#1: | C. Rule#2: |
4. Find the equation of the tangent line to \( f(x) \) at \( x=0 \).
   a. \( f(x) = (4x-3)(x^2+1)^2 \).
   b. \( f(x) = \left[ \cos \left( x - \frac{\pi}{3} \right) \right]^{\frac{1}{x^2+1}} \).

5. A. Find \( \lim_{x \to 0^+} x^2 \ln x \)

B. Find \( \lim_{x \to \infty} (1 + x)^{\frac{3}{x}} \).

6. Liquid is being poured into a cylinder with radius 2cm at a rate of 3.7cm\(^3\)/s. At what rate is the height of the liquid increasing when the volume is \( 16\pi \text{cm}^3 \)?

7. State the behavior of the exponential and natural logarithmic functions (no need to show work):

<table>
<thead>
<tr>
<th>( f(x) = \ln(x) )</th>
<th>( f(x) = e^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA:</td>
<td>HA:</td>
</tr>
<tr>
<td>VA:</td>
<td>VA:</td>
</tr>
<tr>
<td>x-int:</td>
<td>x-int:</td>
</tr>
<tr>
<td>y-int:</td>
<td>y-int:</td>
</tr>
</tbody>
</table>
8. Let \( f(x) = e^{-1/x} \).

A. Find the Domain and Asymptotes  

B. Find x- and y- intercepts  

C. Determine Symmetry  

D. Determine intervals of increase and decrease, and local extrema. Put the information in the table below as shown in class. You may not need the entire table.  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

E. Determine intervals of concavity. Put the information in the table below as shown in class. You may not need the entire table.  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

F. Sketch the graph, based on the above. Plot intercepts, local extrema and inflection points.  

G. Does \( f(x) \) have an absolute max? If so, state where it occurs, and the max value.
9. A. Derive (prove) the formula for the derivative of \( \cos^{-1}x \).

B. Let \( H(x) = \cos^{-1}(1 - 3x) \). Find the derivative of \( H(x) \).

C. Find the domain of \( H(x) \) and of its derivative.

D. Find the limit as \( x \to 0 \) for \( H(x) \) and its derivative.
10. A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

11. Consider the function \( f(x) = (2x - 10)^2 \) on \([0,8]\).
   a. Estimate the area under the curve using \(M_4\). Draw picture and show work.

   ![Graph](image)

   b. Find the exact area by integrating in two different ways.
12. If \( g(x) = \int_{9x^4}^{17} \cos t^5 \, dt \), then find \( g'(x) \).

13. Evaluate
   
   a. \( \int \cos^3 \theta \sin \theta \, d\theta \)
   
   b. \( \int \frac{dx}{5 - 3x} \)
   
   c. \( \int_0^2 x^2 \sqrt{x^3 + 1} \, dx \)
<table>
<thead>
<tr>
<th>Instructor</th>
<th>Section Number</th>
<th>Lecture Time</th>
<th>Activity Time/Office Hours</th>
<th>WebAssign Course Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eldanaf</td>
<td>10,11,12</td>
<td>MW 11-12:15pm</td>
<td>TH 1-4:50pm</td>
<td>csulb</td>
</tr>
<tr>
<td>Hernandez</td>
<td>16,17,18</td>
<td>MW 2-3:15pm</td>
<td>Tu 12-3:50pm</td>
<td>csulb</td>
</tr>
<tr>
<td>English</td>
<td>01,02,03</td>
<td>MW 8-9:15am</td>
<td>F 8-11:50am</td>
<td>csulb</td>
</tr>
<tr>
<td>Viet(x)</td>
<td>31,32,33</td>
<td>TTH 12:30-1:45pm</td>
<td>W 1-4:50pm</td>
<td>csulb</td>
</tr>
<tr>
<td>Hasenjager</td>
<td>37,38,39</td>
<td>TTH 5:30-6:45pm</td>
<td>TTH 7-8:50am</td>
<td>csulb</td>
</tr>
<tr>
<td>Johnson</td>
<td>13,14,15</td>
<td>MW 12-1:45pm</td>
<td>Tu 1-4:50pm</td>
<td>csulb</td>
</tr>
<tr>
<td>Johnson</td>
<td>04,05,06</td>
<td>MW 9:30-10:45pm</td>
<td>F 9-12:50pm</td>
<td>csulb</td>
</tr>
<tr>
<td>Pluta</td>
<td>22,23,24</td>
<td>TTH 9:30-10:45am</td>
<td>W 8-11:50am</td>
<td>csulb</td>
</tr>
<tr>
<td>Segalla</td>
<td>28,29,30</td>
<td>TTH 12:30-1:45pm</td>
<td>W 1-4:50am</td>
<td>csulb</td>
</tr>
<tr>
<td>Kim-Park(x)</td>
<td>19,20,21</td>
<td>TTH 9:30-10:45am</td>
<td>W 9-12:50pm</td>
<td>csulb</td>
</tr>
<tr>
<td>Zhao</td>
<td>07,08,09</td>
<td>MW 9:30-10:45pm</td>
<td>Th 8-11:50pm</td>
<td>csulb</td>
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<td>Chaderjian(x)</td>
<td>25,26,27</td>
<td>TTH 11-12:15pm</td>
<td>F 8-11:50pm</td>
<td>csulb</td>
</tr>
<tr>
<td>Staff</td>
<td>34,35,36</td>
<td>TTH 2-3:15pm</td>
<td>F 8-11:50pm</td>
<td>csulb</td>
</tr>
</tbody>
</table>

HELPFUL WEBSITES