*1. Here is a diagram of the construction of the box. Describe what quantities are changing in the situation when four equal-sized square corners are cut from an 8.5” by 11” sheet of paper.

![Diagram of a box construction]

*2. Create an open box (without a top) by:
   i. Cutting four equal-sized squares from the corners of an 8.5 by 11 inch sheet of paper
   ii. Folding up the sides and taping them together at the edges

   a. Do the cutouts have to be square? Explain.

   b. Use a ruler to measure the length of the side of your square cutout (measured in inches) where the length of the base is the side that was originally 11 inches long and the width of the base is the side that was originally 8.5 inches long.

      Cutout length: _______  Box’s height: _______  Length of box’s base: _______

      Width of the box’s base: ______  Volume of the box: ______

c. Describe a method for computing the box’s volume when all you know is the length of the side of square cutout and you cannot use a ruler.

d. Explain how the length of the base of the box is related to the length of the side of the cutout.
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e. Define a formula to relate the width of the box and the length of the side of the square cutout. Be sure to define your variables.

f. Define a formula to relate the length of the box and the length of the side of the square cutout. Be sure to define your variables.

g. Define a formula to relate the volume of the box to the length of the side of the square cutout.

h. Use your formula from part (g) to represent the volume of the box when \( x \), the length of the side of the cutout, is 0.5 inches.

i. Use your formula from part (g) to represent the volume of the box when \( x \), the length of the side of the cutout, is 3 inches.

3. Represent the volume of the box for cutout sizes of 1.5, 2.7, 3.8, and 4.2 inches using the form above.
   a. When \( x = \) ____ inches, \( V = \) __________ cubic inches
   b. When \( x = \) ____ inches, \( V = \) __________ cubic inches
   c. When \( x = \) ____ inches, \( V = \) __________ cubic inches
   d. When \( x = \) ____ inches, \( V = \) __________ cubic inches
In math we frequently want to refer to the value of a quantity without having to calculate it, and we also want to refer to the rule for calculating a quantity’s value without having to write the rule repeatedly as you did above. The convention we use is called function notation typically expressed “\( f(x) = \text{some rule for processing } x\)”. The diagram below unpacks how all the terms and conventions having to do with functions are packed into function notation.

```

\[
\begin{array}{ccc}
\text{Name} & \text{Input} & \text{Rule} \\
\hline
f & x & (x)(11-2x)(8.5-2x) \\
\end{array}
\]
```

The convention for using function notation is that you write the name of the function, the variable that the rule acts on or takes as input, and then the rule that defines the function. We can use the phrases “name of rule” and “name of function” interchangeably. The values that we put into the rule are called input values. The number that results from applying the rule to an input value is called an output value. The symbol \( f(x) \) is an example of representing the function’s output values for varying values of the input \( x \). The act of using function notation to represent a relationship between two quantities’ values is called defining a function.

Given that the function named \( f \) is defined as above, then

- \( f(0.5) \) represents the box’s volume when the length of the side of the cutout is 0.5 inches. Also, \( f(0.5) \) is the output of \( f \) when given 0.5 as input.
- \( f(x) \) represents the box’s volume when the length of the side of the cutout is \( x \) inches. Notice that \( x \) can vary as the cutout length varies. As \( x \) varies, \( f(x) \) varies too.

We will use function notation repeatedly throughout these modules, so it is important that you are able to read and write relationships between two quantities’ values with this notation.

\*4. Let \( x \) represent the length of the side of the square cutout in inches. Let \( w \) represent the width of the box’s base in inches. Let \( l \) represent the length of the box’s base in inches. Let \( V \) represent the volume of the box in cubic inches.

a. Complete the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( w )</th>
<th>( l )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. What is the smallest possible value for \( x \), the length of the side of the square cutout?
c. What is the largest possible value for $x$, the length of the side of the square cutout?

d. We define the **domain** as the set of all possible input values for our function. What is the domain of the function, $V = f(x) = (x)(11 - 2x)(8.5 - 2x)$?

5. a. View the graphing animation (or use your graphing calculator) to create or view a graph that represents the volume of the box $V$ (measured in cubic inches) in terms of the length of the side of the cutout $x$ (measured in inches). (When determining the window setting on your calculator, consider the possible values of $x$ and the possible values of $V$.) Construct the graph below and label 2 points on the graph. State what each of these points conveys about the box.

![Graph of the function](image)

Point 1: __________________  Point 2: __________________

b. Identify the point on the graph above that corresponds to the dimensions of your box.

c. As $x$ (the length of the side of the cutout) increases from 0.5 to 0.75 inches, how does the volume of the box change?

d. As $x$ (the length of the side of the cutout) increases from 2.1 to 2.7 inches, how does the volume of the box change?
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c. Indicate on the above graph, a change of cutout length from 2 inches to 3 inches. Indicate on the above graph the length that corresponds to the amount the volume changes when the cutout length increases from 2 inches to 3 inches.

f. Estimate the interval(s) of values for the length of the side of the cutout $x$ for which the volume of the box decreases.

*6. Using a graphing calculator, determine the following:
   a. An approximate value for the maximum value of the box.

b. The length of the side of the square cutout when the box has maximum volume.

c. The length of the side of the cutout when the volume of the box is 25 cubic inches.

7. The expression $(11 - 2x)(8.5 - 2x)$ calculates the area of a box’s base when the box is made with a square cutout of length $x$.
   a. Use the letter $k$ to name a function that has the length of the square cutout as input (measured in inches) and the area of the corresponding box’s base as output (measured in square inches). Define the base’s area as a function of the length of the square cutout. Name the parts of the function’s definition.

b. Using function notation, represent the area of the box’s base when the length of the square cutout is 0.2 inches, 2.7 inches, and 4.1 inches. (Do not calculate the volume; represent the volume determined by each length of the square cutout.)
c. The use of letters is arbitrary when defining functions, as long as you define what they represent. Use any letter to represent the length of the square cutout and use another letter to name the area of the box’s base. Define the area of the box’s base as a function of the length of the square cutout. When defining the letters used to represent the quantities, describe the quantity measured and the unit of measurement for the quantity.

d. Compare your definition with others in your group. Did you define the same function using different letters?

8. Evaluate each of the following:
   a. \( f(7) \) when \( f(x) = \frac{x^2 + (-2 + x)}{7x - 8} \).

   b. \( g(-5.1) \) when \( g(x) = \frac{-2x^2 + 1.5x - 3(-x + 9)}{8.1 - 4x} \).

   c. \( g(m) \) when \( g(x) = \frac{2}{3}x - 6x^3 + x(x + 3) \).

   *d. \( f(x + 2) \) when \( f(x) = 2x^2 + 8x - 12 \).