Homework 9 Help
Questions

Multiple Choice Questions:

1. Suppose the marginal product of labor is 8 and the marginal product of capital is 2. If the wage rate is $4 and the price of capital is $2, then in order to minimize costs the firm should use
   a. More capital and less labor
   b. More labor and less capital
   c. Three times more capital than labor
   d. None of the statements associated with this question are correct

2. You are an efficiency expert hired by a manufacturing firm that uses K and L as inputs. The firm produces and sells a given output. If \( w = 40 \), \( r = 100 \), \( MPP_L = 20 \), and \( MPP_K = 40 \) the firm:
   a. Is cost minimizing
   b. Should use less L and more K to cost minimize
   c. Should use more L and less K to cost minimize
   d. Is profit maximizing but not cost minimizing

3. In order to minimize the cost of producing a given level of output, a firm manager should use more inputs when:
   a. Its price rises
   b. Its price falls
   c. Its price remains the same
   d. The price of other inputs fall

4. Suppose the marginal product of labor is 10 and the marginal product of capital is 8. If the wage rate is $5 and the price of capital is $2, then in order to minimize costs the firm should use
   a. More capital and less labor
   b. More labor and less capital
   c. Equal amounts of labor and capital
   d. None of the statements associated with this question are correct

5. A firm produces output according to a production function: \( F(L,K) = 2L + 4K \) If the wage rate is $10 per hour and the rental rate on capital is $5 per hour, what is the cost-minimizing input mix for producing 100 units of output?
   a. \( L = 50; K = 25 \)
   b. \( L = 0; K = 25 \)
   c. \( L = 50; K = 0 \)
   d. \( L = 25; K = 50 \)

6. Suppose your production function is given by: \( F(L,K)=Q=\min\{2K,2L\} \). The wage rate is \( w=2 \) and the price of capital is \( r=3 \). What is the profit-maximizing level of L and K to produce Q=20 units of output?
   a. \( L=10, K=10 \)
   b. \( L=0, K=10 \)
   c. \( L=10; K=0 \)
   d. \( L=10; K=5 \)
Problem Solving

**Question 1**
Suppose that the production function of a firm is given as:

\[ F(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}} \]

\( w = 54; r = 2; Q = 60 \). You are in the long-run.

a. Find the profit-maximizing/cost-minimizing quantity of labor \((L)\) and capital \((K)\).

b. Calculate the cost of producing \( Q = 60 \) units of output.

**Question 2**
Suppose that you are in the long run and the production function of a firm is given as:

\[ F(L, K) = L^{\frac{1}{2}} + K^{\frac{1}{2}} \]

\( w = 2; \quad r = 2; \quad Q = 50 \).

a. Find the profit-maximizing/cost-minimizing quantity of labor \((L)\) and capital \((K)\).

b. Calculate the cost of producing \( Q = 50 \) units of output when \( w = 2 \) and \( r = 2 \).

c. Now suppose the price of capital doubles. Find the profit-maximizing/cost-minimizing quantity of labor \((L)\) and capital \((K)\).

d. Calculate the cost of producing \( Q = 50 \) units of output when \( w = 2 \) and \( r = 4 \).

**Question 3**
Suppose that you are in the long-run and the production function of a firm is given as:

\[ F(L, K) = L^{\frac{1}{2}} + K \]

\( w = 1; \quad r = 2; \quad Q = 30 \).

a. Find the profit-maximizing/cost-minimizing quantity of labor \((L)\) and capital \((K)\). Also compute the cost of producing \( Q = 30 \) units of output when \( w = 1 \) and \( r = 2 \).

b. Now suppose that technological improvements change the production function to: \( F(L, K) = 2L^{\frac{1}{2}} + K \). Find the profit-maximizing/cost-minimizing quantity of labor \((L)\) and capital \((K)\). Also compute the cost of producing \( Q = 30 \) units of output with the new production function.

c. Now suppose that technology is the same as in part (a), i.e., \( F(L, K) = L^{\frac{1}{2}} + K \). Suppose that instead of \( Q = 30 \), you need to increase your output to \( Q = 40 \). Find the profit-maximizing/cost-minimizing quantity of labor \((L)\) and capital \((K)\). Also compute the cost of producing \( Q = 40 \) units of output.

**Question 4**
Suppose that the production function for your firm is given as:

\[ F(L, K) = K^{\frac{1}{2}} L^{\frac{1}{2}} \]

a. Suppose that in the short-run, capital is fixed at 1 unit. If the firm can sell its output at a price of \$216 per unit, \( w = 2 \) and \( r = 6 \), how many units of labor should this firm hire in order to maximize profits?

b. How many units of output does the firm produce in the short-run?

c. What are the profits in the short-run?

d. Assuming that in the long-run both capital \((K)\) and labor \((L)\) are variable inputs, what is the optimal combination (profit-maximizing/cost-minimizing \( L^* \) and \( K^* \)) to produce the amount of output you found in part (b)?

e. What are the profits in the long-run?
**Question 5**
Suppose that the production function for your firm is given as:

\[ F(L, K) = L^{\frac{1}{2}} + K^{\frac{1}{2}} \]

a. Suppose that in the **short-run**, labor is fixed at 16 unit. If the firm can sell its output at a price of $400 per unit, \( w = 10 \) and \( r = 20 \), how many units of labor should this firm hire in order to maximize profits/minimize costs?
b. How many units of output does the firm produce in the **short-run**?
c. What are the profits in the **short-run**?
d. Assuming that in the **long-run** both capital (K) and labor (L) are variable inputs, what is the optimal combination (profit-maximizing/cost-minimizing) to produce the amount of output you found in part (b)?
e. What are the profits in the **long-run**?

**Question 6**
Suppose that the following production function represents a firm's ability to manufacture pencils:

\[ F(L, K) = 2K^{\frac{1}{2}}L^{\frac{1}{2}} \]

a. Assume that the wage rate is equal to 1 and the rental rate on capital is equal to 4. Find the optimal combination of Labor (L) and capital (K) to produce 40 units of output. Find the cost of producing this output (ie find the variable cost function).
b. In addition assume that the firm has a fixed cost of production equal to 16. Find the variable cost, the total cost, the marginal cost, and the average cost of producing 40 units of output.

**Question 7**
Suppose that the production function for your firm is given as:

\[ Q = F(L, K) = L^{\frac{1}{2}} + K^{\frac{1}{2}} \]

and that \( w = 16 \), \( r = 2 \).
a. Suppose that you are in the long-run and that both labor and capital are variable inputs. Find the optimal combination of labor and capital in order to produce \( Q \) units of output.
b. In addition, assume that the firm has a fixed cost of $200. Find the **variable cost** function, the **total cost** function, the **marginal cost** function, the **average total cost** function, the **average fixed cost** function, and the **average variable cost** function.

**Question 8**
Suppose that the production function for your firm is given as:

\[ Q = F(L, K) = K^{\frac{1}{2}}L^{\frac{1}{2}} \]

and that \( w = 2 \), \( r = 4 \).
a. Suppose that you are in the long-run and that both labor and capital are variable inputs. Find the optimal combination of labor and capital in order to produce \( Q \) units of output.
b. Write the variable cost function.
c. In addition, assume that the firm has a fixed cost of $100. Find the **total cost** function, the **marginal cost** function, the **average total cost** function, the **average fixed cost** function, and the **average variable cost** function.